

# The Magic of Four Zero Neutrino Yukawa Textures

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## TALK PLAN

- Four zero textures and Motivation
- $\mu\tau$  symmetry, tribimaximal mixing and their effects
- $l_\alpha \rightarrow l_\beta \gamma$  and leptogenesis
- Phenomenology
- Deviations from RG running
- Conclusion

## Four zero textures and Motivation

Type-I See-Saw

Lagrangian mass term

$$-\mathcal{L}^m = \bar{l}_L(M_\ell)_{ll'} l'_R + \bar{\nu}_{lL}(m_D)_{ll'} N_{l'R} + \frac{1}{2} \bar{N}_{lL}^c (M_R)_{ll'} N_{l'R} + h.c.$$

Neutrino mass matrix

$$-\mathcal{L}_\nu^m = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{N}_L^c \end{pmatrix}_l \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}_{ll'} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix}_{l'} + h.c.$$

for  $O(M_R) \gg O(m_D) \implies$

$$-\mathcal{L}_{\nu_L}^m = \frac{1}{2} \bar{\nu}_{lL}(M_\nu)_{ll'} \nu_{lR}^c + h.c.$$

$M_\nu \simeq -m_D M_R^{-1} m_D^T$  (See – Saw formula) .

Without loss of generality choose basis: Diagonal  $M_\ell$  and  $M_R$  with real positive entries.

$M_\nu$  Diagonalisation:

$$U^\dagger M_\nu U^* = M_d^\nu = \text{diag}(m_1, m_2, m_3)$$

$m_i$ 's Real Positive.

Relation of flavor basis to mass basis:  $(\nu_L)_l = (U)_{li} \nu_{iL}$

Charged current interaction in mass basis:

$$\mathcal{L}_l^{cc} = (-g/\sqrt{2}) \bar{l}_{iL} \gamma^\mu U_{ij} \nu_{Lj} W_\mu^-$$

$U$  Parametrization:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & -s_{13}e^{-i\delta_D} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_D} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_D} & -s_{23}c_{13} \\ -s_{12}s_{23} + c_{12}c_{23}s_{13}e^{i\delta_D} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta_D} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\alpha_M} & 0 & 0 \\ 0 & e^{i\beta_M} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

= PMNS  $\times$  Majorana Phase Matrix

$$c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij}$$

$\delta_D$  = Dirac phase

$\alpha_M, \beta_M$  = Majorana phases

In our chosen basis :

$$M_\ell = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \quad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$$

and in this basis

$$m_D = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

with complex elements.

$M_\nu$  from See-Saw

$$M_\nu = -m_D M_R^{-1} m_D^T = - \begin{pmatrix} \frac{a_1^2}{M_1} + \frac{a_2^2}{M_2} + \frac{a_3^2}{M_3} & \frac{a_1 b_1}{M_1} + \frac{a_2 b_2}{M_2} + \frac{a_3 b_3}{M_3} & \frac{a_1 c_1}{M_1} + \frac{a_2 c_2}{M_2} + \frac{a_3 c_3}{M_3} \\ \frac{a_1 b_1}{M_1} + \frac{a_2 b_2}{M_2} + \frac{a_3 b_3}{M_3} & \frac{b_1^2}{M_1} + \frac{b_2^2}{M_2} + \frac{b_3^2}{M_3} & \frac{b_1 c_1}{M_1} + \frac{b_2 c_2}{M_2} + \frac{b_3 c_3}{M_3} \\ \frac{a_1 c_1}{M_1} + \frac{a_2 c_2}{M_2} + \frac{a_3 c_3}{M_3} & \frac{b_1 c_1}{M_1} + \frac{b_2 c_2}{M_2} + \frac{b_3 c_3}{M_3} & \frac{c_1^2}{M_1} + \frac{c_2^2}{M_2} + \frac{c_3^2}{M_3} \end{pmatrix}$$

## Experimental Facts:

- $|R| = \frac{\Delta m_{21}^2}{|\Delta m_{32}^2|} \simeq 3.2 \times 10^{-2}$
- $\theta_{23} \simeq \frac{\pi}{4}$ ,     $\theta_{12} \simeq \sin^{-1} \frac{1}{\sqrt{3}}$ ,     $\theta_{13}$  small
- No family of neutrino is decoupled

Additional inputs:  $m_i \neq 0$     and    all  $M_i$ 's are large (like  $10^9$  Gev or more)

$$\det M_\nu \neq 0 \neq \det m_D$$

Texture zeros assumed in the Lagrangian, i.e. in  $m_D$

Maximum number of zeros allowed in  $m_D$  with above constraints = 4

(G.C. Branco, D.E.-Costa, M.N. Rebelo and P. Roy, *PRD* 08)

All four zero textures have been classified. 72 allowed textures

One important property of these:

High Scale CP Violation completely determined by the Low Energy CP Violation

72 textures categorised into two

Category A: 54 Textures:

- 18 Textures, First row orthogonal to second row,  $(M_\nu)_{12} = (M_\nu)_{21} = 0$
- 18 Textures, First row orthogonal to third row,  $(M_\nu)_{13} = (M_\nu)_{31} = 0$
- 18 Textures, Second row orthogonal to third row,  $(M_\nu)_{23} = (M_\nu)_{32} = 0$

## Category B: Textures with two zeros in one row and one each in the rest

- 6 Textures, First row with two zeros
- 6 Textures, Second row with two zeros
- 6 Textures, Third row with two zeros

If the rows with one zero each are  $k, l$  then  $\det \text{cofactor } [(M_\nu)_{kl}] = 0$

In addition for each textures in B:  $(M_\nu)_{ll'} \neq 0$

## $\mu\tau$ symmetry, tribimaximal mixing and the consequences

Under  $\mu\tau$  symmetry,

$$\nu_\mu \longleftrightarrow \nu_\tau, \quad N_\mu \longleftrightarrow N_\tau : \quad m_D = \begin{pmatrix} a_1 & a_2 & a_2 \\ b_1 & b_2 & b_3 \\ b_1 & b_3 & b_2 \end{pmatrix}, \quad M_\nu = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}.$$

Custodial  $\mu\tau$  symmetry in  $M_\nu$ :

$$M_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} \Rightarrow \quad h = M_\nu M_\nu^\dagger = \begin{pmatrix} P & Q & Q \\ Q^* & R & S \\ Q^* & S & R \end{pmatrix}$$

where

$$\begin{aligned} P &= |A|^2 + 2|B|^2, & Q &= A^*B + B^*(C + D) \\ R &= |B|^2 + |C|^2 + |D|^2, & S &= |B|^2 + CD^* + DC^*. \end{aligned}$$

Apply to four zero textures.

## Category A:

- $\mu\tau$  symmetry in Lagrangian is incompatible with only 4 zeros in  $m_D$  for 52 textures here
- In the remaining 2 textures  $\mu\tau$  symmetry can be fitted

$$m_D^{(1)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & b_2 & 0 \\ 0 & 0 & b_2 \end{pmatrix}, \quad m_D^{(2)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix} \quad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$$

See-Saw formula  $\Rightarrow$

$$M_\nu = - \begin{pmatrix} a_1^2/M_1 + 2a_2^2/M_2 & a_2b_2/M_2 & a_2b_2/M_2 \\ a_2b_2/M_2 & b_2^2/M_2 & 0 \\ a_2b_2/M_2 & 0 & b_2^2/M_2 \end{pmatrix},$$

for both.

## Under parametrization

$$m_3 = -\frac{b_2^2}{M_2}, \quad \sqrt{\frac{M_2}{M_1}} \times \frac{a_1}{b_2} = k_1 e^{i(\alpha+\alpha')} \quad \frac{a_2}{b_2} = k_2 e^{i\alpha'}$$

$M_\nu$  after phase factor absorption  $e^{i\alpha'}$  by  $\nu_e$  :

$$M_\nu = m_3 \begin{pmatrix} k_1^2 e^{2i\alpha} + 2k_2^2 & k_2 & k_2 \\ k_2 & 1 & 0 \\ k_2 & 0 & 1 \end{pmatrix}.$$

3 real parameters  $k_1, k_2, \alpha$

Further, tribimaximal mixing assumption, i.e.  $\theta_{12} = \sin^{-1}(1/\sqrt{3}) \Rightarrow$

$$A + B = C + D$$

$$\Rightarrow \quad \alpha = \pi/2 \quad k_1 = (2k_2^2 + k_2 - 1)^{1/2}.$$

One real parameter.

## Category B:

Again, 16 textures here with only four zeros in  $m_D$  are incompatible with  $\mu\tau$  symmetry.  
Hence ruled out

In remaining two textures  $\mu\tau$  Symmetry imposed without ambiguity

$$m_D = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ b_1 & 0 & b_2 \end{pmatrix}, \quad \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix} \quad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$$

See-Saw formula  $\Rightarrow$

$$M_\nu = - \begin{pmatrix} a_1^2/M_1 & a_1 b_1/M_1 & a_1 b_1/M_1 \\ a_1 b_1/M_1 & b_1^2/M_1 + b_2^2/M_2 & b_1^2/M_1 \\ a_1 b_1/M_1 & b_1^2/M_1 & b_1^2/M_1 + b_2^2/M_2 \end{pmatrix},$$

for both.

Under parametrization

$$m_3 = -\frac{b_2^2}{M_2}, \quad \sqrt{\frac{M_2}{M_1}} \times \frac{a_1}{b_2} = l_1 e^{i\beta'} \quad \sqrt{\frac{M_2}{M_1}} \times \frac{b_1}{b_2} = l_2 e^{i\beta}$$

Absorbing phase factor  $e^{i\beta'}$  in  $\nu_e$ :

$$M_\nu = m_3 \begin{pmatrix} l_1^2 & l_1 l_2 e^{i\beta} & l_1 l_2 e^{i\beta} \\ l_1 l_2 e^{i\beta} & l_2^2 e^{2i\beta} + 1 & l_2^2 e^{2i\beta} \\ l_1 l_2 e^{i\beta} & l_2^2 e^{2i\beta} & l_2^2 e^{2i\beta} + 1 \end{pmatrix}.$$

Again, 3 real parameters  $l_1, l_2, \beta$ .

Now, additional TBM assumption  $\theta_{12} = \sin^{-1}(1/\sqrt{3}) \Rightarrow$

$$\begin{aligned} \beta &= \cos^{-1}(l_1/4l_2), \\ l_2 &= \frac{1}{2}(1 - l_1^2)^{1/2}. \end{aligned}$$

One real parameter.

## Phenomenology

Diagonlisation of  $\mu\tau$  symmetric  $h = M_\nu M_\nu^\dagger \Rightarrow$

$$\tan 2\theta_{12} = \frac{2\sqrt{2}|Q|}{R + S - P} \quad \text{along with } \theta_{23} = \frac{\pi}{4}, \theta_{13} = 0$$

Solar mass squared difference

$$\Delta m_{21}^2 = m_2^2 - m_1^2 = \sqrt{(R + S - P)^2 + 8|Q|^2}$$

Atmospheric mass squared difference

$$\Delta m_{32}^2 = m_3^2 - m_2^2 = \frac{R - 3S - P - \sqrt{(R + S - P)^2 + 8|Q|^2}}{2}$$

$> 0$	(normal mass ordering)
$< 0$	(inverted mass ordering)

## Category A with $\mu\tau$ symmetry

$$\tan 2\theta_{12} = \frac{X_1}{X_2}$$

$$R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{2(X_1^2 + X_2^2)^{1/2}}{X_3 - (X_1^2 + X_2^2)^{1/2}}$$

where,

$$X_1 = 2\sqrt{2}k_2\sqrt{(2k_2^2 + 1)^2 + 2k_1^2(2k_2^2 + 1)\cos 2\alpha + k_1^4}$$

$$X_2 = 1 - 4k_2^4 - k_1^4 - 4k_1^2k_2^2\cos 2\alpha$$

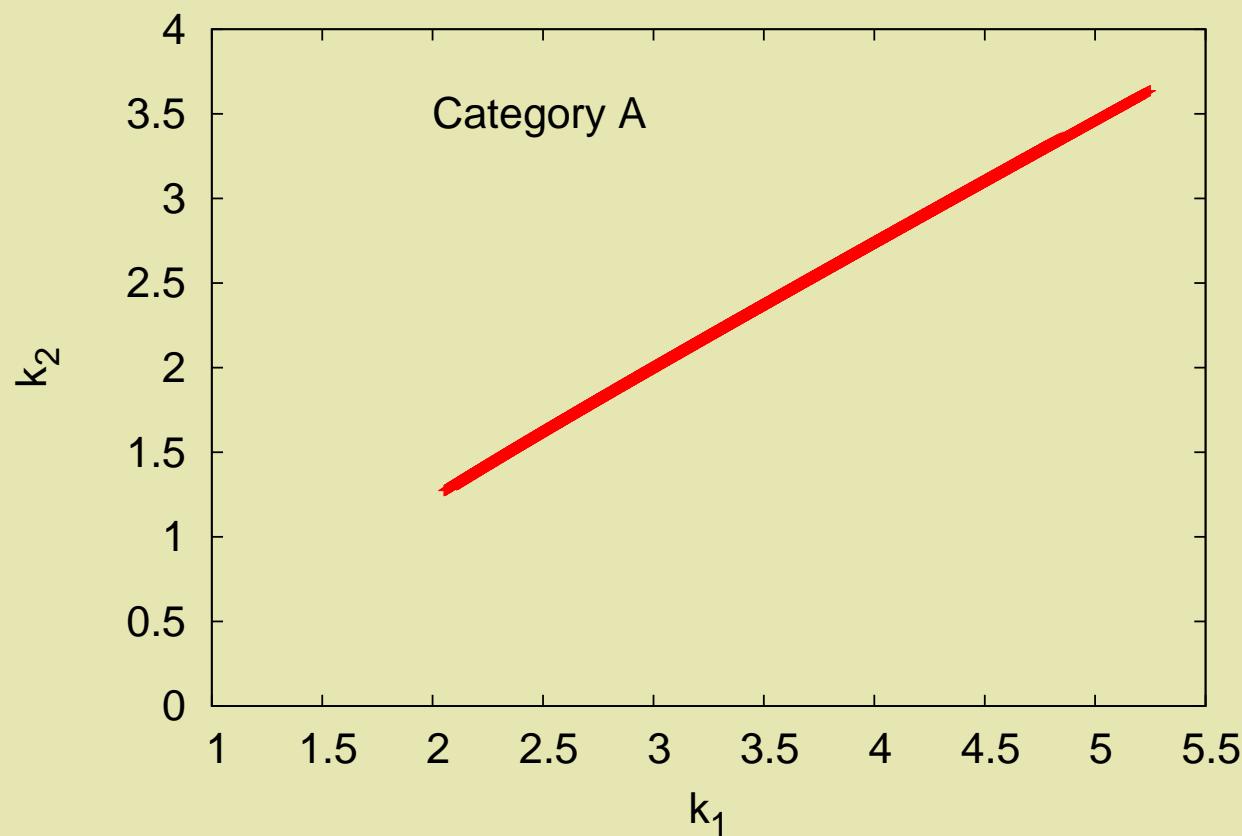
$$X_3 = 1 - 4k_2^4 - k_1^4 - 4k_1^2k_2^2\cos 2\alpha - 4k_2^2.$$

$$\text{TBM} \implies R = \frac{3(k_2 - 2)}{k_2 + 1}$$

Only inverted ordered mass spectrum is allowed in this category: a result originally due to Merle, Rodejohann, PRD06

There is no common parameter space ( $k_1$ ,  $k_2$ ,  $\alpha$ ) for  $1\sigma$  range of  $R \times 10^2 = -(2.88 - 3.37)$  and  $\theta_{12} = 33.15^\circ - 35.91^\circ$

3D allowed common parameter space ( $k_1$ ,  $k_2$ ,  $\alpha$ ): For  $R \times 10^2 = -(2.46 - 3.99)(3\sigma)$  and  $\theta_{12} = 30.66^\circ - 39.23^\circ(3\sigma)$  shown in projected  $(k_1, k_2)$  plane.



## Restrictions on parameter space

- $89^\circ \leq \alpha \leq 90^\circ$
- $2.0 < k_1 < 5.3$
- $1.2 < k_2 < 3.7$
- Only inverted mass ordering allowed

TBM  $\implies 1.95 \leq k_2 \leq 1.97$  at the  $3\sigma$  level

Category B:

$\mu\tau$  symmetry again  $\Rightarrow$

$$\tan 2\theta_{12} = \frac{X_1}{X_2}$$

$$R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{2(X_1^2 + X_2^2)^{1/2}}{X_3 - (X_1^2 + X_2^2)^{1/2}}$$

where,

$$X_1 = 2\sqrt{2k_2}\sqrt{(2l_2^2 + l_1^2)^2 + 2(2l_2^2 + l_1^2)\cos 2\beta + 1}$$

$$X_2 = 4l_2^4 - l_1^4 + 4l_2^2 \cos 2\beta + 1$$

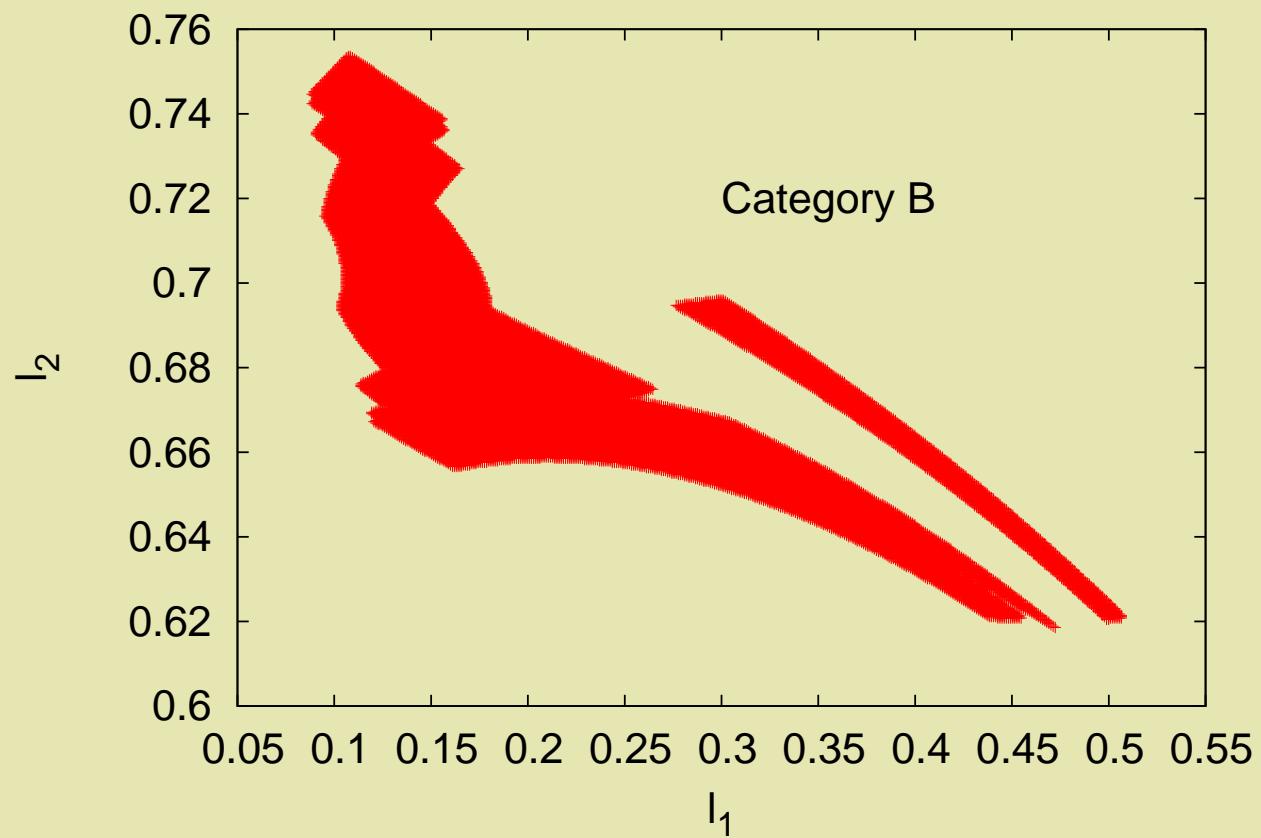
$$X_3 = 1 - (2l_2^2 + l_1^2)^2 - 4l_2^2 \cos 2\beta.$$

TBM  $\Rightarrow$

$$R = \frac{3l_1^2}{2 - 4l_1^2}$$

Only normal ordered mass spectrum is allowed in this category

3D allowed parameter space ( $l_1$ ,  $l_2$ ,  $\beta$ ): For  $R \times 10^2 = 2.52 - 4.07(3\sigma)$  and  $\theta_{12} = 30.66^\circ - 39.23^\circ (3\sigma)$  Shown in projected  $(l_1, l_2)$  plane



## Restrictions on parameter space

- $87^\circ \leq \beta \leq 90^\circ$
- $0.1 < l_1 < 0.55$
- $0.6 < l_2 < 0.76$
- Only normal ordered mass spectrum allowed

TBM  $\implies 0.11 \leq l_1 \leq 0.15$  at the  $3\sigma$  level

## Radiative lepton decay: $l_\alpha \rightarrow l_\beta \gamma$

$$l_\alpha \rightarrow l_\beta \gamma \quad \alpha > \beta : l_1 = e, \quad l_2 = \mu, \quad l_3 = \tau$$

mSUGRA scenarios with universal scalar masses at high scale  $M_X (\sim 2 \times 10^{16} \text{ GeV}) \Rightarrow$

$$\text{BR}(l_\alpha \rightarrow l_\beta \gamma) \propto \text{BR}(l_\alpha \rightarrow l_\beta \nu \bar{\nu}) \left| (m_D)_{\alpha i} (m_D)_{\beta i}^* \ln \frac{M_X}{M_i} \right|$$

$M_i$  = mass of i-th heavy right chiral neutrino

Category A:

Allowed two textures  $\rightarrow (m_\nu)_{23} = 0 \Rightarrow \text{BR}(\tau \rightarrow \mu \gamma) = 0$

In both Category A and Category B:

$(m_\nu)_{13} = (m_\nu)_{12} \neq 0 \Rightarrow \text{BR}(\tau \rightarrow e \gamma) \neq 0 \neq \text{BR}(\mu \rightarrow e \gamma)$

$$\frac{\text{BR}(\tau \rightarrow e \gamma)}{\text{BR}(\mu \rightarrow e \gamma)} \simeq \frac{\text{BR}(\tau \rightarrow e \nu_e \bar{\nu}_e)}{\text{BR}(\mu \rightarrow e \nu_\mu \bar{\nu}_\mu)} \simeq 0.178.$$

## Leptogenesis

Flavor dependent lepton asymmetry:

$$\begin{aligned}\epsilon_i^\alpha &\equiv \frac{\Gamma(N_i \rightarrow \phi \bar{l}_\alpha) - \Gamma(N_i \rightarrow \phi^\dagger l_\alpha)}{\sum_\beta [\Gamma(N_i \rightarrow \phi \bar{l}_\beta) + \Gamma(N_i \rightarrow \phi^\dagger l_\beta)]} \\ &\propto \sum_{j \neq i} \left[ \mathcal{I}_{ij}^\alpha f \left( \frac{M_j^2}{M_i^2} \right) + \mathcal{J}_{ij}^\alpha \left( 1 - \frac{M_j^2}{M_i^2} \right)^{-1} \right] + O(M_W^2/M_i^2)\end{aligned}$$

$$\mathcal{I}_{ij}^\alpha = \text{Im}(m_D^\dagger)_{i\alpha} (m_D)_{\alpha j} (m_D^\dagger m_D)_{ij} = -\mathcal{I}_{ji}^\alpha$$

$$\mathcal{J}_{ij}^\alpha = \text{Im}(m_D^\dagger)_{i\alpha} (m_D)_{\alpha j} (m_D^\dagger m_D)_{ji} = -\mathcal{J}_{ji}^\alpha$$

$$f(x) = \sqrt{x} \left[ \frac{2}{1-x} - \ln \frac{1+x}{x} \right] \quad (\text{MSSM})$$

## Flavor independent leptogenesis

$$\epsilon_i = \sum_{\alpha} \epsilon_i^{\alpha} \propto \sum_{j \neq i} [(m_D^\dagger m_D)_{ij}]^2 f(M_j^2/M_i^2)$$

For  $M_1 \ll M_{2,3}$ ,  $f(M_{2,3}^2/M_1^2) \neq -3M_1/M_{2,3}$

Effective mass for washout of a flavor asymmetry

$$\tilde{m}_1^{\alpha} = |(m_D)_{\alpha 1}|^2 / M_1$$

configuration	$I_{ij}^{\alpha}$	$J_{ij}^{\alpha}$	$\tilde{m}_1^e$	$\tilde{m}_1^{\mu}$	$\tilde{m}_1^{\tau}$
$m_D^{(1)}$	$I_{12}^e = \mathcal{I}_{13}^e \neq 0$ , rest zero	0	nonzero	0	0
$m_D^{(2)}$	— do —	0	nonzero	0	0
$m_D^{(3)}$	$I_{12}^{\mu} = \mathcal{I}_{13}^{\tau} \neq 0$ , rest zero	0	nonzero	nonzero	equals $\tilde{m}_1^{\mu}$
$m_D^{(4)}$	$I_{13}^{\mu} = \mathcal{I}_{12}^{\tau} \neq 0$ , rest zero	0	nonzero	nonzero	equals $\tilde{m}_1^{\mu}$

## Deviations from RG

$\mu\tau$  symmetry imposed at  $\Lambda \sim 10^{12}$  GeV

One loop RG running from  $\Lambda$  to  $\lambda \sim 10^3$  GeV → deviations

In MSSM using  $m_\tau^2 \gg m_{e,\mu}^2$  deviation  $\Delta_\tau$ :

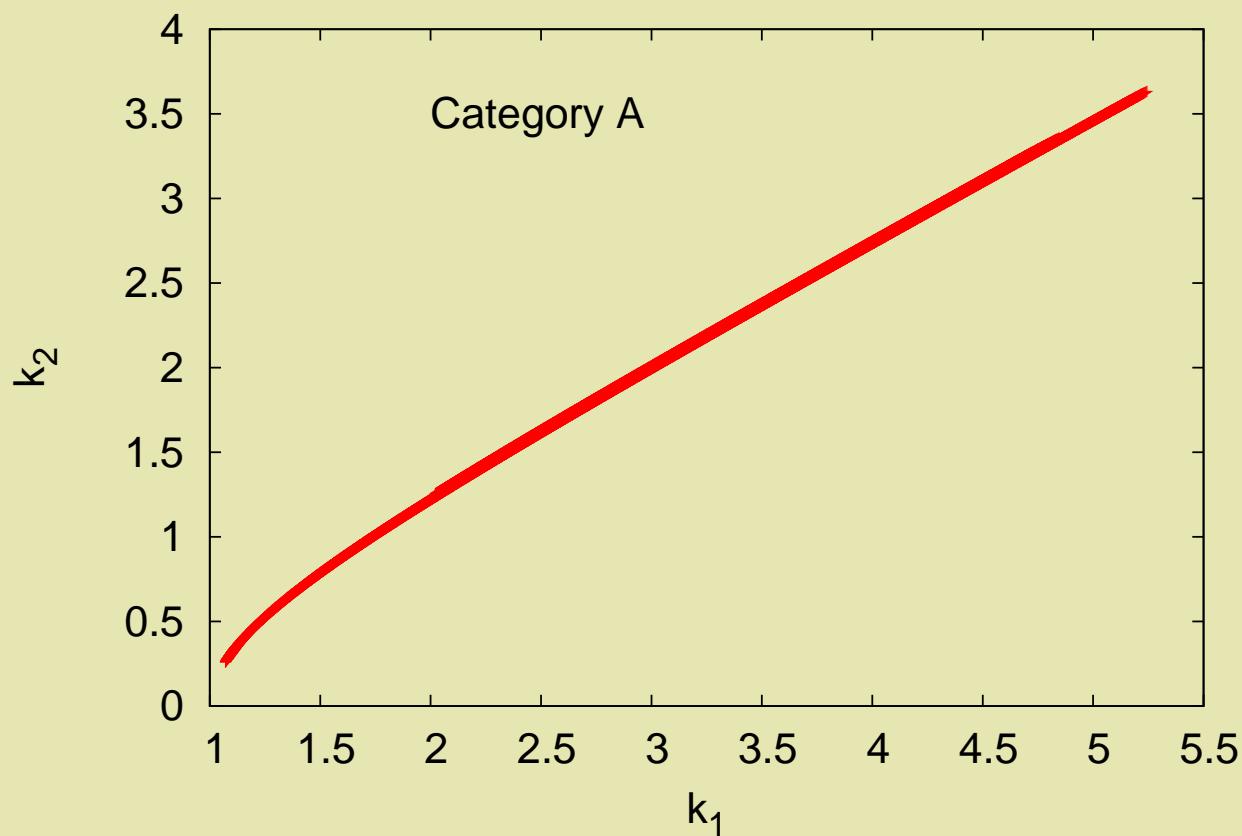
$$\Delta_\tau \simeq \frac{m_\tau^2}{8\pi^2 v^2} (\tan^2 \beta + 1) \ln \left( \frac{\Lambda}{\lambda} \right),$$

$$v^2 = 2(v_u^2 + v_d^2), \quad \tan \beta = \frac{v_u}{v_d}.$$

Now

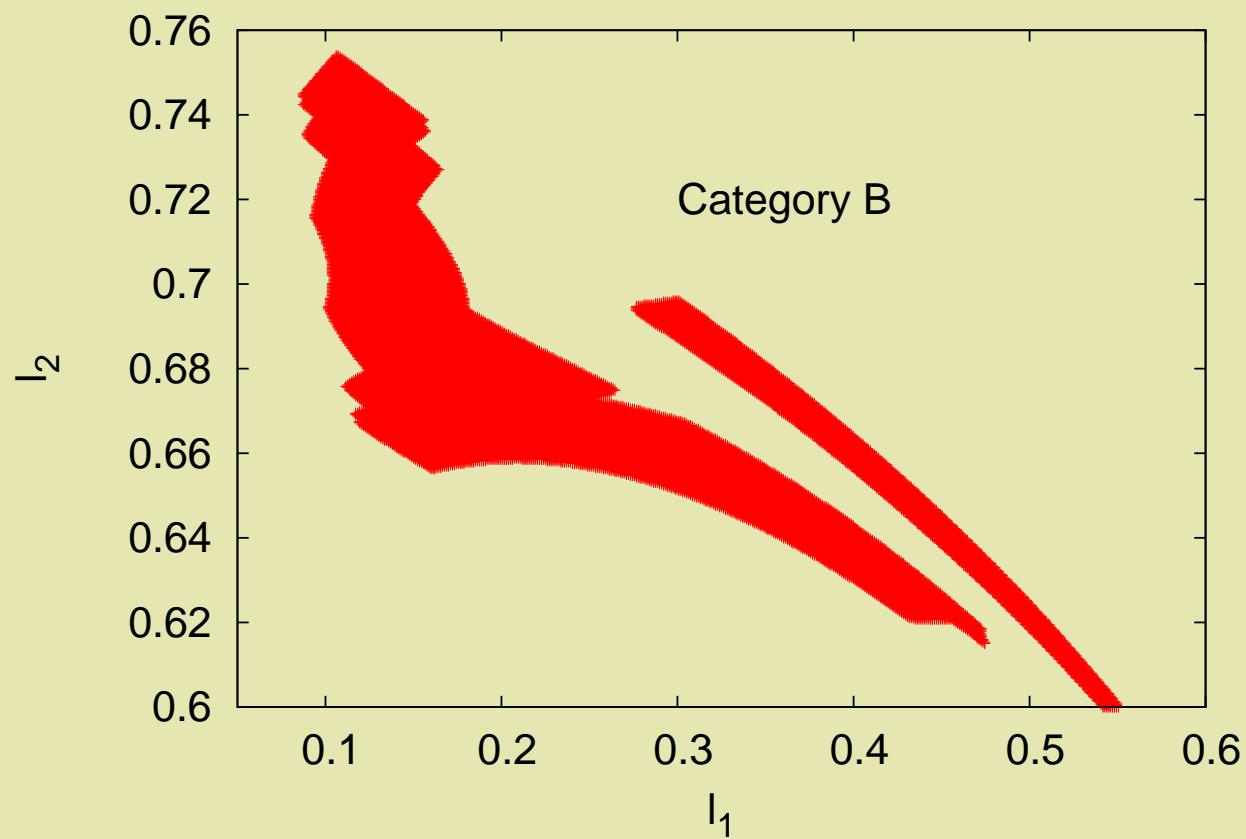
$$m_\nu^\lambda = \begin{pmatrix} A & B & B(1 - \Delta_\tau) \\ B & C & D(1 - \Delta_\tau) \\ B(1 - \Delta_\tau) & D(1 - \Delta_\tau) & C(1 - 2\Delta_\tau) \end{pmatrix}$$

Deviations affect the results in both categories but marginally. New parameter space are shown in figures.



Results in category A for  $3\sigma$  variations of  $R$  and  $\theta_{12}$

- $0^\circ \leq \theta_{13}^\lambda \leq 2.7^\circ$
- $\theta_{23}^\lambda \leq 45^\circ$
- Inverted mass ordering retained



## Results in category B for $3\sigma$ variations of $R$ and $\theta_{12}$

- $0^\circ \leq \theta_{13}^\lambda \leq 0.85^\circ$
- $\theta_{23}^\lambda \geq 45^\circ$
- Normal mass ordering retained

## Conclusion

- Out of 126 four zero textures  $\mu\tau$  symmetry compatible with FOUR textures only leading to only two forms of  $M_\nu$ : one for category A, one for category B
- For these  $\theta_{12}$  and  $R$  admit restricted regions in the parameter space and  $M_\nu^{(A)}$  is in some tension with data
- Tri-bimaximal mixing further highly restricts the parameters
- Small radiative deviations from  $\mu\tau$  yield rather small  $\theta_{13}$  and further restrict  $\theta_{23}$